Mathematics Methods
Unit 4

Continuous random variable - Normal distribution

1. Normal distribution

Definition: Normal distribution (also known as the Gaussian distribution or the bell curve) is a continuous probability distribution wherein values lie in a symmetrical fashion mostly situated around the mean.

Examples of continuous random variable that are exactly or approximately normal:

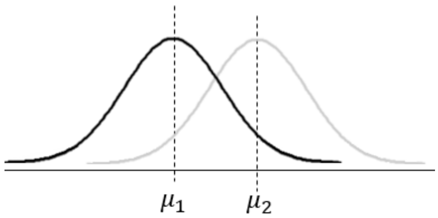
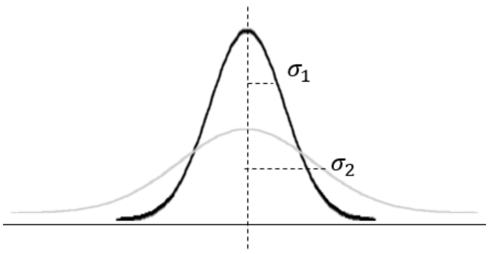
- Blood pressure
- Measurement error
- IQ scores
- Height

Probability density function of normal distribution

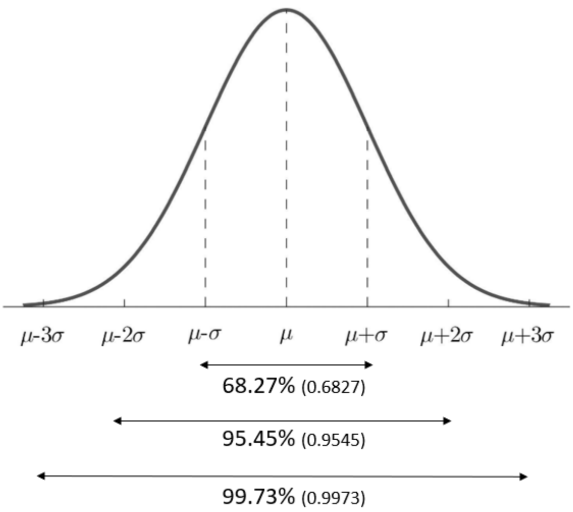
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}, -\infty < x < \infty$$

Basic properties of normal distribution:

- It is symmetric about the mean
- The mean = the mode = the median
- The curve is unimodal (one peak), maximum point at $(\mu, \frac{1}{\sigma\sqrt{2\pi}})$
- The curve approaches but never touches, the x-axis, as it extends farther and farther away from the mean ($-\infty < x < \infty$)
- Total area under the curve = 1 ($\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}\, dx = 1$)
- Mean, μ and standard deviation, σ

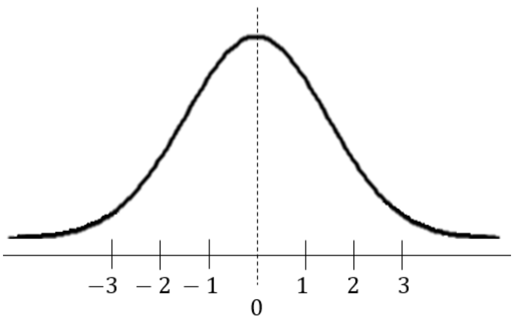
Mean, μ	The larger the mean, the greater the shift towards the right  $\mu_2 > \mu_1$
Standard deviation, σ	The larger the standard deviation, the greater the vertical compression  $\sigma_2 > \sigma_1$

Area under normal distribution and its corresponding standard deviation away from mean, μ.



2. Standard normal distribution

Definition: Standard normal distribution is a normal distribution with mean equals to 0 while standard deviation equals to 1.

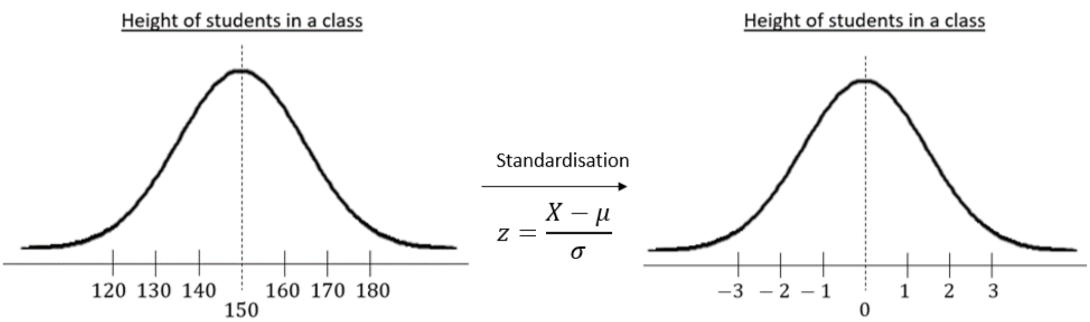


Standardisation formula:

$$z = \frac{X - \mu}{\sigma}$$

Basic properties of standard normal distribution: (same as normal distribution)
z score tells the number of standard deviation, σ is the value away from the mean, μ

Example:



Normal distribution

Standard normal distribution

3.	**Calculating the probability of normal distribution**
	(a) Given the X score
	Example 1: Find the probability of a test score less than 20% given that the test score is normally distributed $X \sim (50, 10^2)$. Example 2: The length of screws in the toolbox is normally distributed with mean of 1 cm and standard deviation of 0.05 cm. find the probability that a randomly selected screw exceeds 1.1 cm.
	(b) Given the percentage/ standard deviation away from mean
	Example: A set of normally distributed chisels has mean of 2 cm and standard deviation of 0.01 cm. Find the probability that a chisel picked at random is two standard deviation away from mean.
4.	**Finding the mean/ standard deviation/ X score**
	(a) Finding the mean and standard deviation
	Example 1: Given that X is a normal distribution that has mean, μ and variance of 20 as well that $P(X > 60) = 0.02235$ find the value of mean.

Example 2:
A random variable T has a normal distribution with mean of 39 and variance, σ^2. Given that $P(X > 42.5) = 0.098876$, find the standard deviation.

Example 3:
The books in a library follows a normal distribution with mean, μ and variance 0.14. Given that $P(X < 20) = 0.0342$, find the mean.

Example 4:
Given that $X \sim N(\mu, \sigma^2)$, $P(10 < X < 20) = 0.234$ and $P(X < 10) = 0.0992$.

END